ANNEXE 3 : DIAGRAMMES : EFFORTS TRANCHANTS - MOMENTS FLÉCHISSANTS

(Version du 23 juillet 2023 (13h15))

Les diagrammes qui suivent sont extraient de "*Steel Designers' Manuel*" Fourth Edition Metric, may 1972, Crosby Lockwood Staples - London.

Notations :

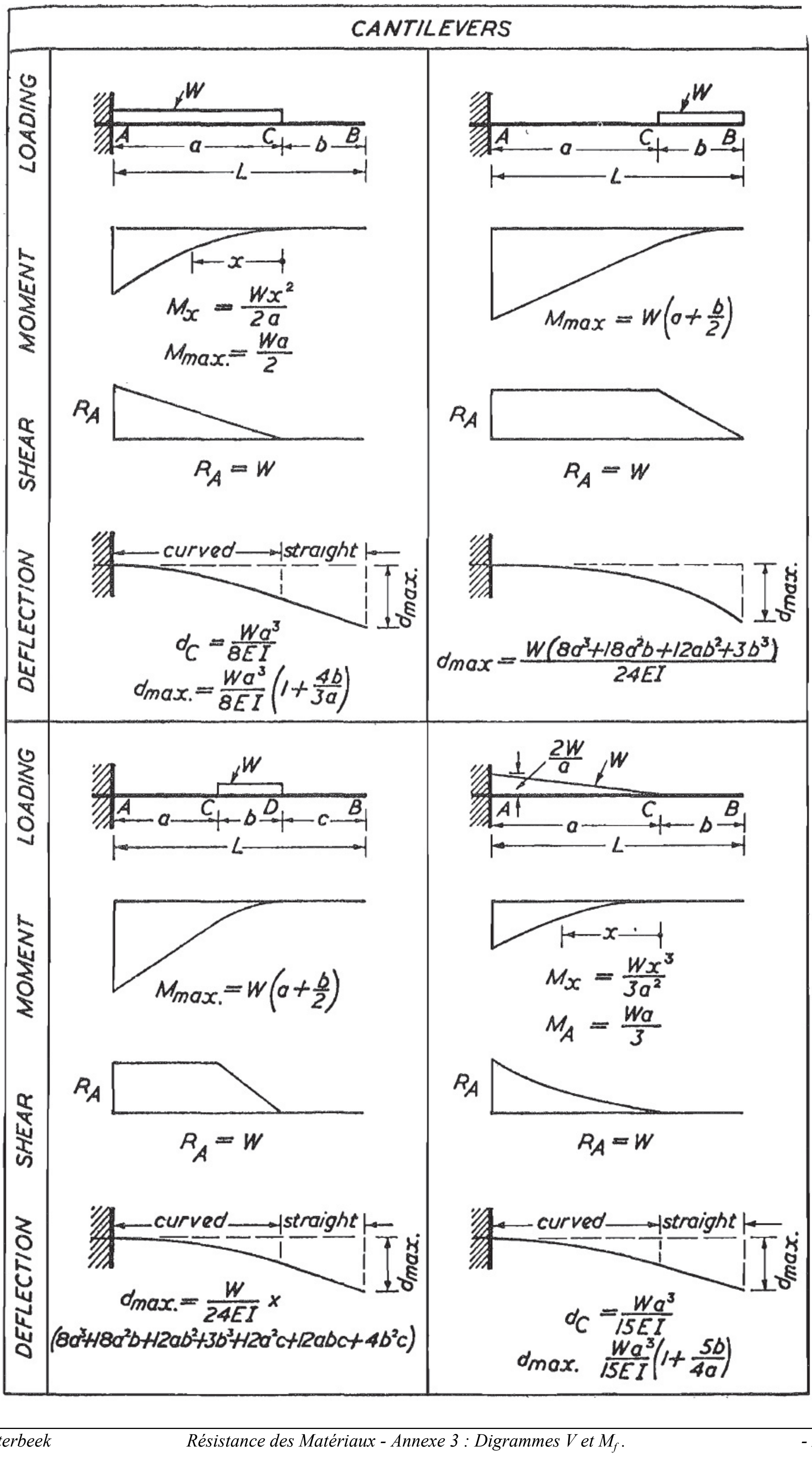
a, b, c	longueur (partielle)	*mm*
L	longueur totale de la poutre	*mm*
M_x	moment fléchissant en *x*	*Nmm*
M_{max}	moment fléchissant maximum	*Nmm*
R_A, R_B	réaction d'appui en A, B	*N*
d_C	flèche (déformation) en C	*M*
d_{max}	flèche (déformation) maximale	*M*
E	module de Young	N/mm^2
I	moment d'inertie (de la poutre)	mm^4
W	effort en *N* correspondant à la charge répartie par mètre courant *p* multiplié par la *longueur* totale chargée	*N*
w	effort en *N* par *mm* (charge répartie)	*N/mm*
P	effort (ponctuel)	*N*

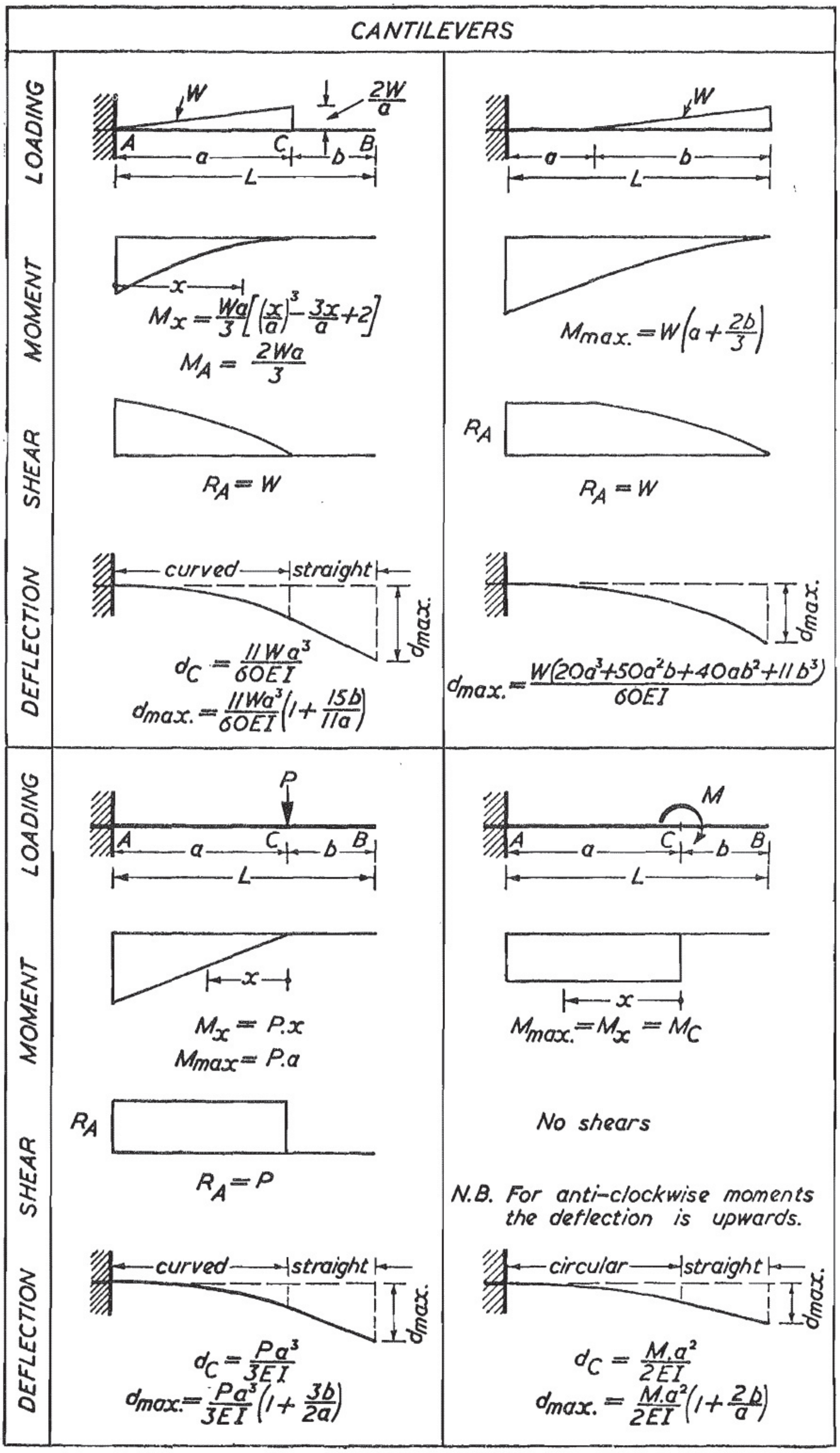
Traduction :

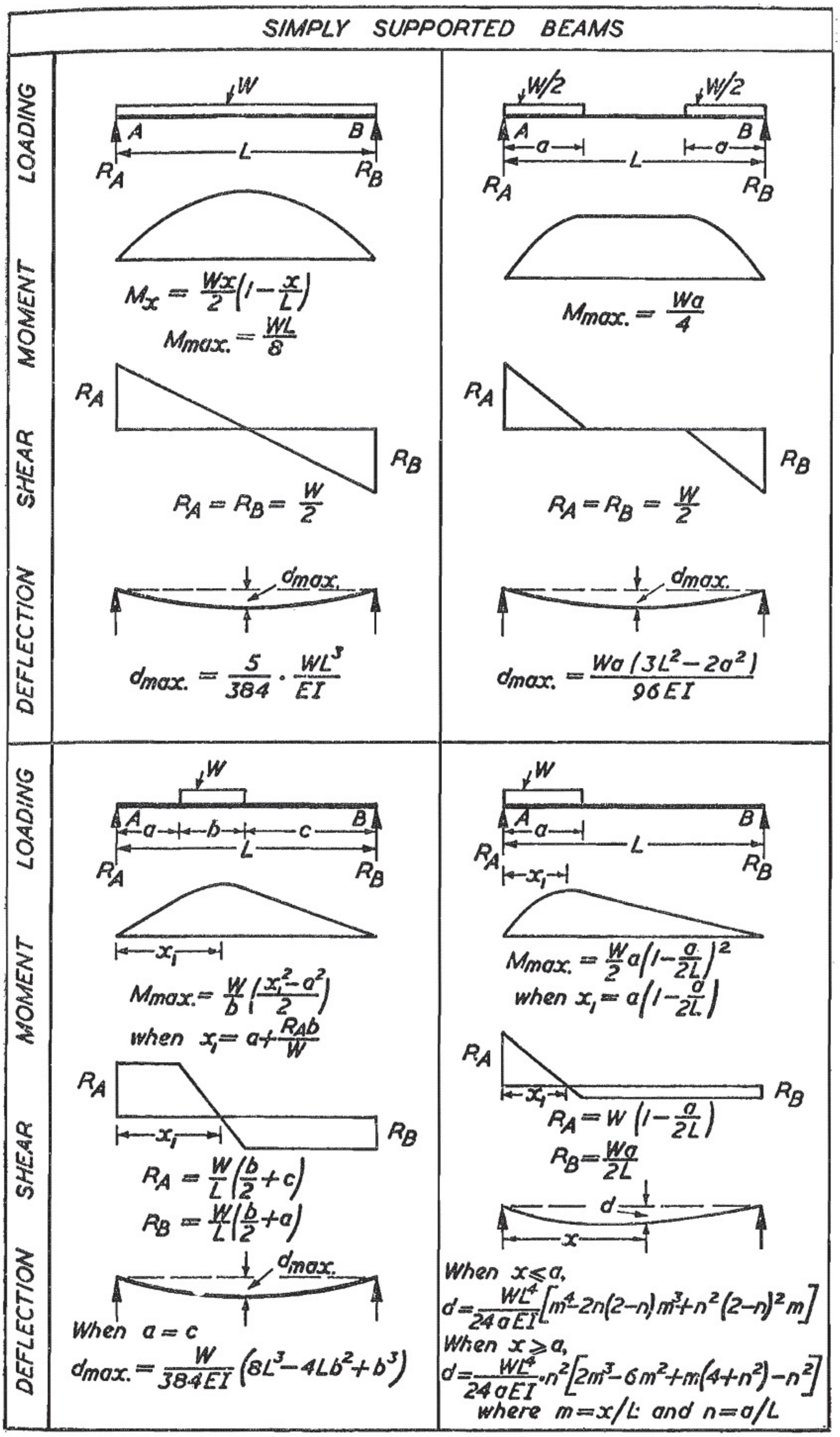
Loading	*Chargement*
Moment	*Moment (fléchissant)*
Shear	*Effort tranchant*
Deflection	*Déformation*
Cantilevers	*(Poutre en) porte-à-faux*
Curved	*(En) courbe (point de vue déformation)*
Straight	*Droit (point de vue déformation)*

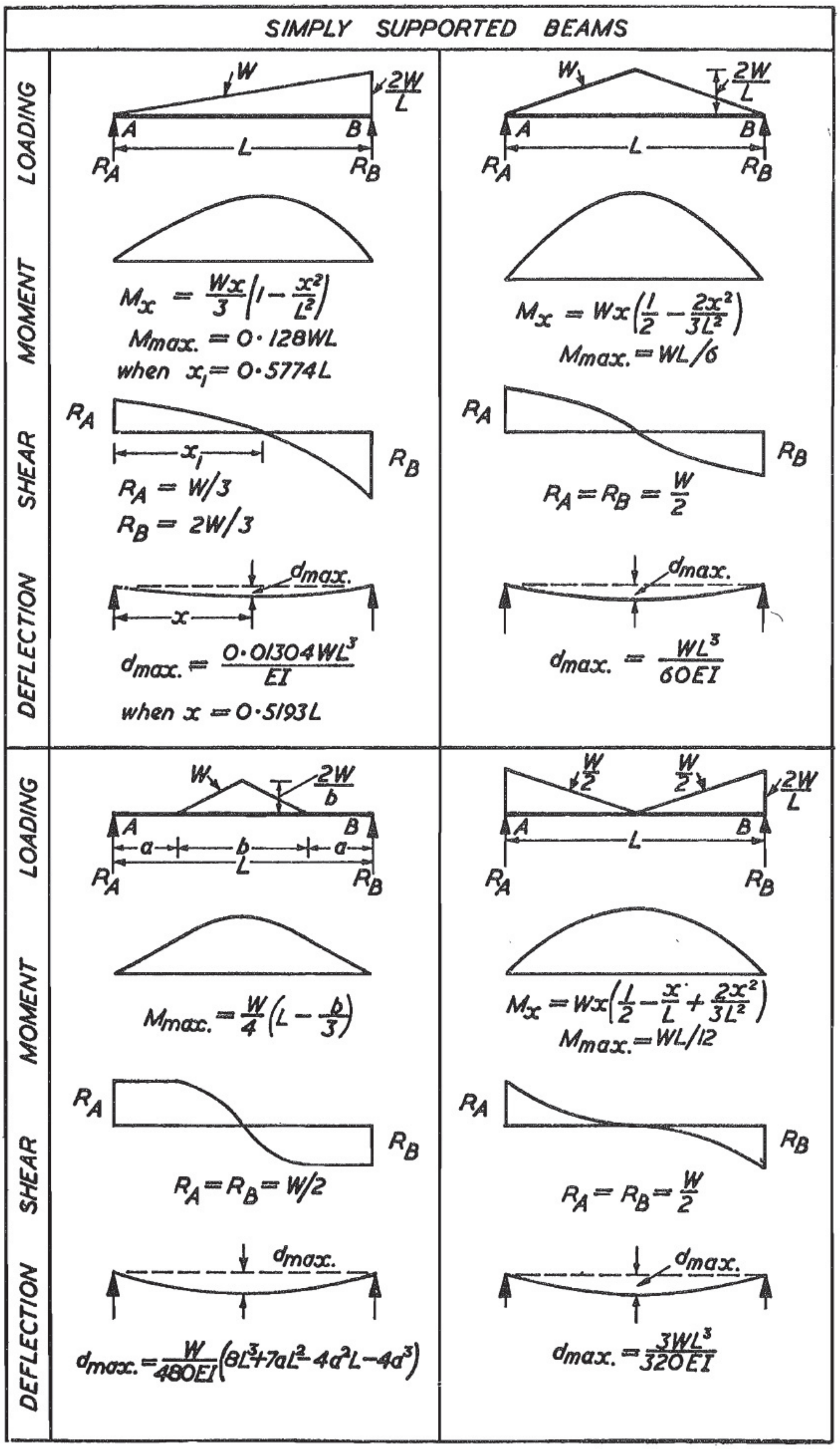
Remarque :

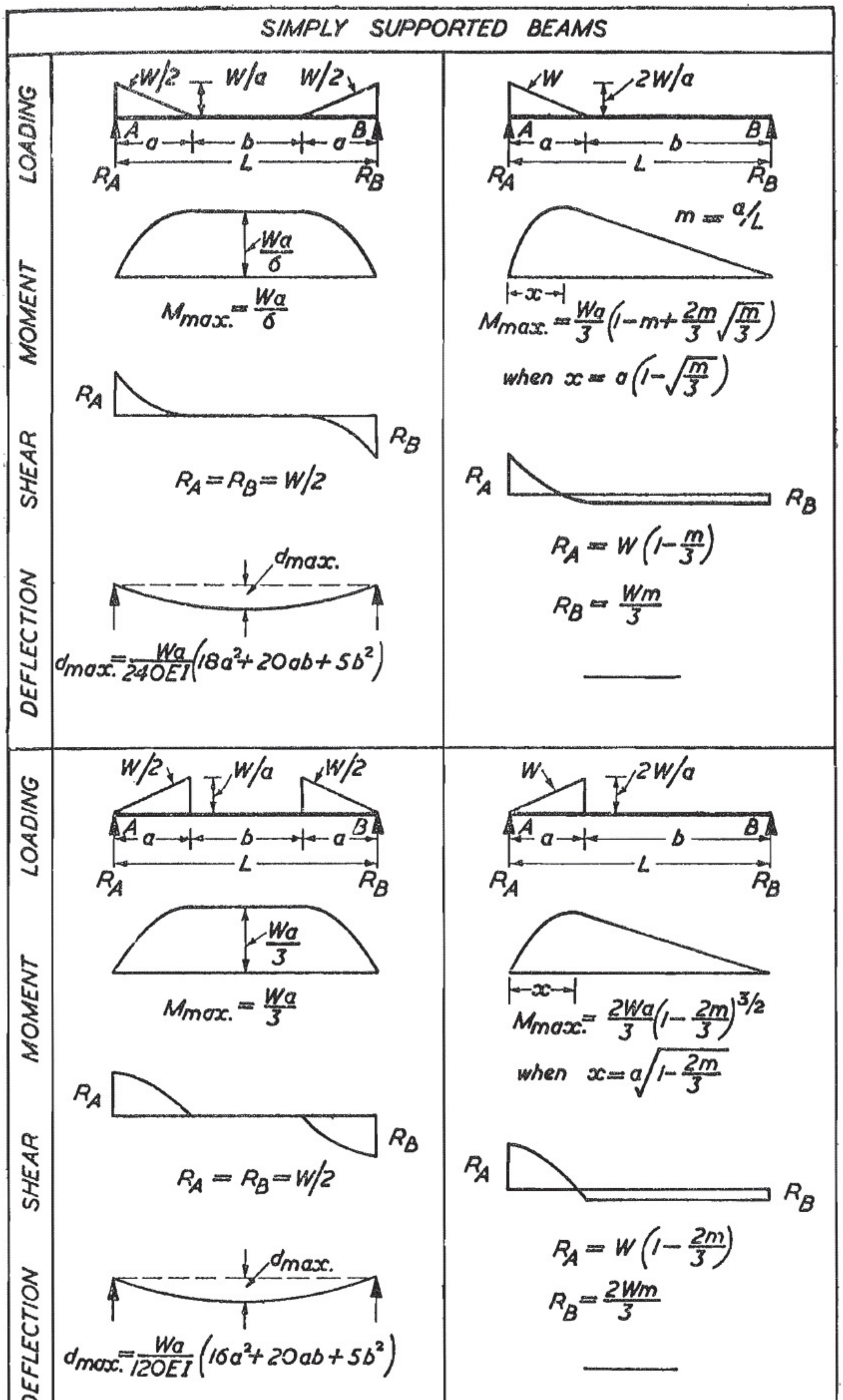
Concernant les divers diagrammes qui suivent, en général, pour celui des Moments fléchissants et des Efforts tranchants, la *convention est inverse* de celle que nous avons prise, c'est-à-dire "+" au dessus et "-" en dessous de la ligne du "0".
Sauf pour les poutres "cantilevers" où pour le diagramme des Moment fléchissants, uniquement, est "+" en dessous et "-" au dessus (notre convention à nous).

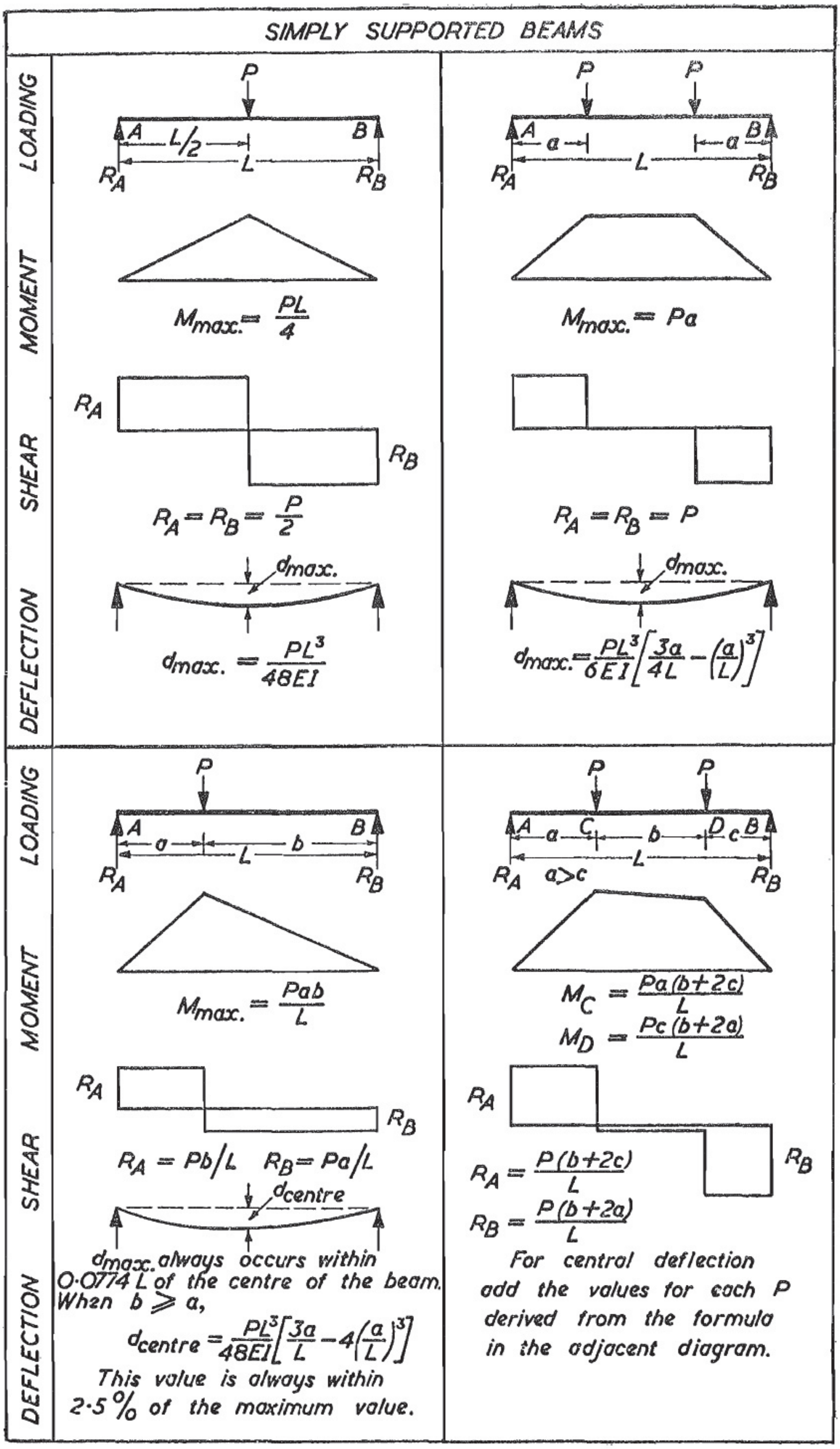

CANTILEVERS
LOADING
MOMENT
SHEAR
DEFLECTION
W
A a C b B
L
x
Mx = Wx²/2a
Mmax. = Wa/2
RA
RA = W
curved straight
dmax.
dC = Wa³/8EI
dmax. = Wa³/8EI (1 + 4b/3a)
Mmax = W(a + b/2)
dmax = W(8a³+18a²b+12ab²+3b³)/24EI
A a C b D c B
Mmax. = W(a + b/2)
dmax. = W/24EI x (8a³+18a²b+12ab²+3b³+12a²c+12abc+4b²c)
2W/a
Mx = Wx³/3a²
MA = Wa/3
dC = Wa³/15EI
dmax. Wa³/15EI (1 + 5b/4a)

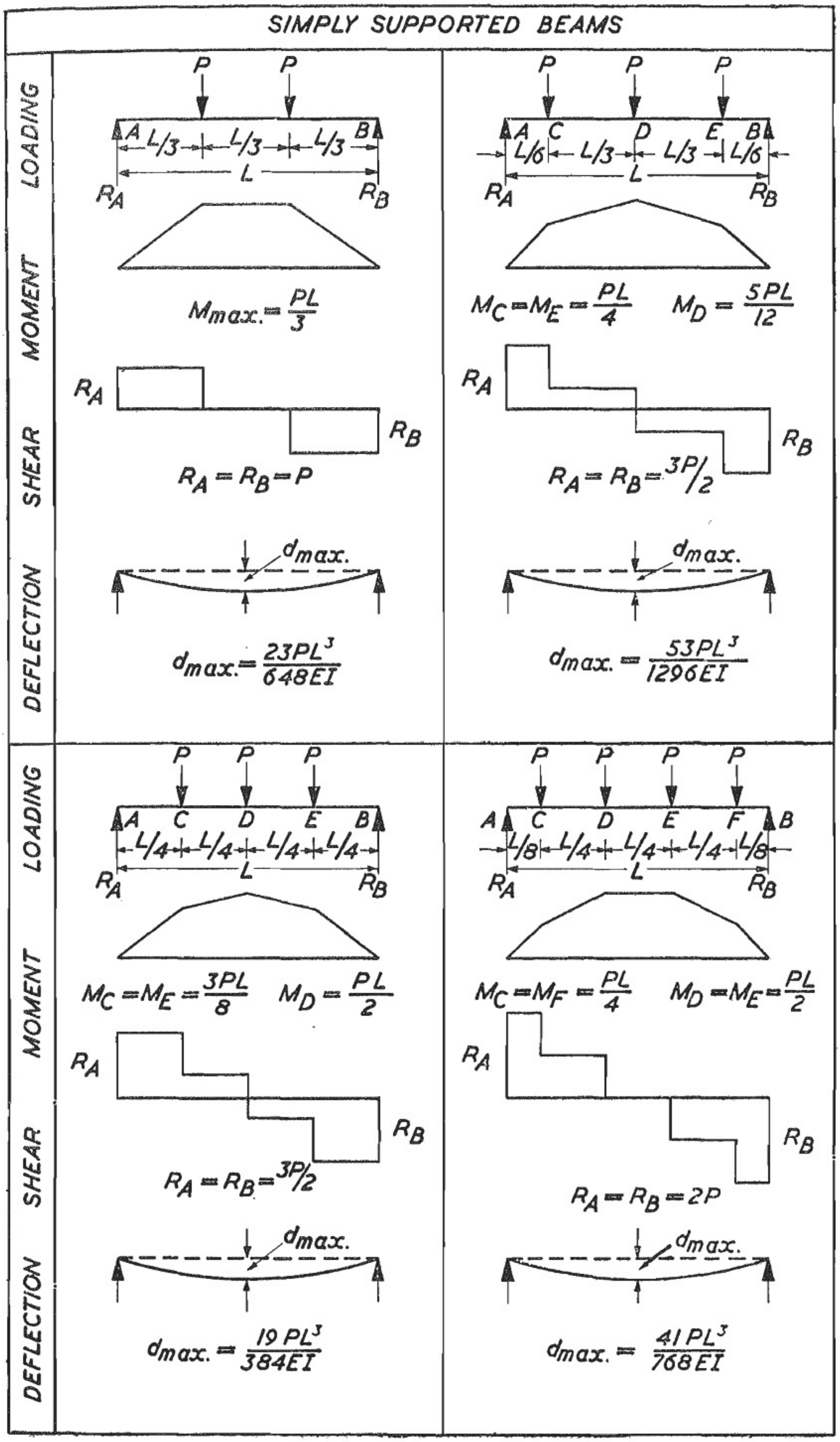

CANTILEVERS
LOADING
MOMENT
SHEAR
DEFLECTION
W
2W/a
A
C
B
a
b
L
x
Mx = Wa/3 [(x/a)³ - 3x/a + 2]
MA = 2Wa/3
RA = W
curved
straight
dmax.
dC = 11Wa³/60EI
dmax. = 11Wa³/60EI (1 + 15b/11a)
Mmax. = W(a + 2b/3)
RA
RA = W
dmax. = W(20a³+50a²b+40ab²+11b³)/60EI
P
Mx = P.x
Mmax = P.a
RA = P
dC = Pa³/3EI
dmax. = Pa³/3EI (1 + 3b/2a)
M
Mmax. = Mx = MC
No shears
N.B. For anti-clockwise moments the deflection is upwards.
circular
dC = M.a²/2EI
dmax. = M.a²/2EI (1 + 2b/a)


SIMPLY SUPPORTED BEAMS
LOADING
MOMENT
SHEAR
DEFLECTION
W
A
B
L
R_A
R_B
M_x = Wx/2 (1 − x/L)
M_max. = WL/8
R_A = R_B = W/2
d_max.
d_max. = 5/384 · WL^3/EI
W/2
W/2
a
a
M_max. = Wa/4
R_A = R_B = W/2
d_max. = Wa(3L^2 − 2a^2)/96EI
a
b
c
x_1
M_max. = W/b ((x_1^2 − a^2)/2)
when x_1 = a + R_A b/W
R_A = W/L (b/2 + c)
R_B = W/L (b/2 + a)
When a = c
d_max. = W/384EI (8L^3 − 4Lb^2 + b^3)
M_max. = W/2 a(1 − a/2L)^2
when x_1 = a(1 − a/2L)
R_A = W(1 − a/2L)
R_B = Wa/2L
d
x
When x ≤ a,
d = WL^4/24aEI [m^4 − 2n(2 − n)m^3 + n^2(2 − n)^2 m]
When x ≥ a,
d = WL^4/24aEI · n^2 [2m^3 − 6m^2 + m(4 + n^2) − n^2]
where m = x/L and n = a/L


SIMPLY SUPPORTED BEAMS
LOADING
MOMENT
SHEAR
DEFLECTION
W
2W/L
A
B
L
R_A
R_B
M_x = (Wx/3)(1 − x²/L²)
Mmax. = 0·128WL
when x₁ = 0·5774L
x₁
R_A = W/3
R_B = 2W/3
dmax.
x
dmax. = 0·01304WL³/EI
when x = 0·5193L
M_x = Wx(1/2 − 2x²/3L²)
Mmax. = WL/6
R_A = R_B = W/2
dmax. = WL³/60EI
2W/b
a
b
Mmax. = (W/4)(L − b/3)
R_A = R_B = W/2
dmax. = (W/480EI)(8L³ + 7aL² − 4a²L − 4a³)
W/2
M_x = Wx(1/2 − x/L + 2x²/3L²)
Mmax. = WL/12
dmax. = 3WL³/320EI

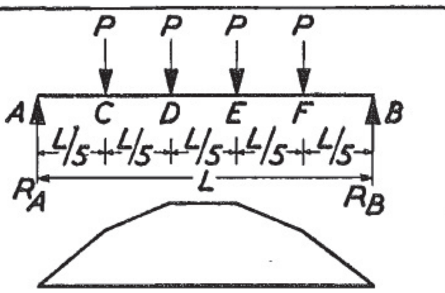

SIMPLY SUPPORTED BEAMS
LOADING
MOMENT
SHEAR
DEFLECTION
W/2
W/a
W/2
A
B
a
b
a
L
R_A
R_B
Wa/6
M_max. = Wa/6
R_A = R_B = W/2
d_max.
d_max. = Wa/240EI (18a² + 20ab + 5b²)
W
2W/a
m = a/L
x
M_max. = Wa/3 (1 − m + 2m/3 √(m/3))
when x = a(1 − √(m/3))
R_A = W(1 − m/3)
R_B = Wm/3
Wa/3
M_max. = Wa/3
d_max. = Wa/120EI (16a² + 20ab + 5b²)
M_max. = 2Wa/3 (1 − 2m/3)^{3/2}
when x = a√(1 − 2m/3)
R_A = W(1 − 2m/3)
R_B = 2Wm/3


SIMPLY SUPPORTED BEAMS
LOADING
MOMENT
SHEAR
DEFLECTION
P
A
B
L/2
L
R_A
R_B
$M_{max.} = \frac{PL}{4}$
$R_A = R_B = \frac{P}{2}$
$d_{max.}$
$d_{max.} = \frac{PL^3}{48EI}$
P P
a
$M_{max.} = Pa$
$R_A = R_B = P$
$d_{max.} = \frac{PL^3}{6EI}\left[\frac{3a}{4L} - \left(\frac{a}{L}\right)^3\right]$
a
b
$M_{max.} = \frac{Pab}{L}$
$R_A = Pb/L \quad R_B = Pa/L$
d_{centre}
$d_{max.}$ always occurs within
0·0774 L of the centre of the beam.
When $b \geqslant a$,
$d_{centre} = \frac{PL^3}{48EI}\left[\frac{3a}{L} - 4\left(\frac{a}{L}\right)^3\right]$
This value is always within
2·5 % of the maximum value.
C
D
c
a>c
$M_C = \frac{Pa(b+2c)}{L}$
$M_D = \frac{Pc(b+2a)}{L}$
$R_A = \frac{P(b+2c)}{L}$
$R_B = \frac{P(b+2a)}{L}$
For central deflection
add the values for each P
derived from the formula
in the adjacent diagram.

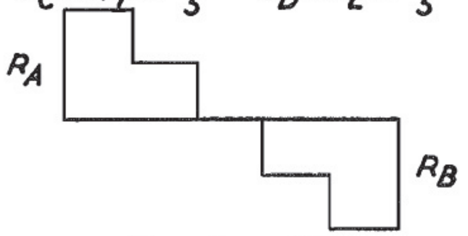

SIMPLY SUPPORTED BEAMS
LOADING
MOMENT
SHEAR
DEFLECTION
P P
A L/3 L/3 L/3 B
L
R_A R_B
$M_{max.} = \frac{PL}{3}$
$R_A = R_B = P$
$d_{max.}$
$d_{max.} = \frac{23PL^3}{648EI}$
P P P
A C D E B
L/6 L/3 L/3 L/6
L
R_A R_B
$M_C = M_E = \frac{PL}{4}$ $M_D = \frac{5PL}{12}$
$R_A = R_B = 3P/2$
$d_{max.}$
$d_{max.} = \frac{53PL^3}{1296EI}$
P P P
A C D E B
L/4 L/4 L/4 L/4
L
R_A R_B
$M_C = M_E = \frac{3PL}{8}$ $M_D = \frac{PL}{2}$
$R_A = R_B = 3P/2$
$d_{max.}$
$d_{max.} = \frac{19PL^3}{384EI}$
P P P P
A C D E F B
L/8 L/4 L/4 L/4 L/8
L
R_A R_B
$M_C = M_F = \frac{PL}{4}$ $M_D = M_E = \frac{PL}{2}$
$R_A = R_B = 2P$
$d_{max.}$
$d_{max.} = \frac{41PL^3}{768EI}$

SIMPLY SUPPORTED BEAMS

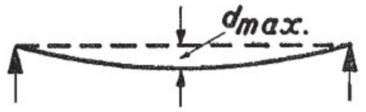
LOADING / MOMENT / SHEAR / DEFLECTION



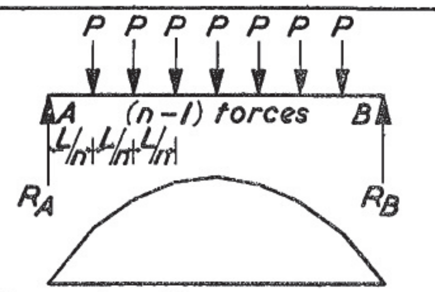
$M_C = M_F = \frac{2PL}{5}$ $M_D = M_E = \frac{3PL}{5}$



$R_A = R_B = 2P$



$$d_{max.} = \frac{63PL^3}{1000EI}$$



When n is odd,

$$M_{max.} = \frac{(n^2-1)\,PL}{8n}$$

When n is even,

$$M_{max.} = n.PL/8$$

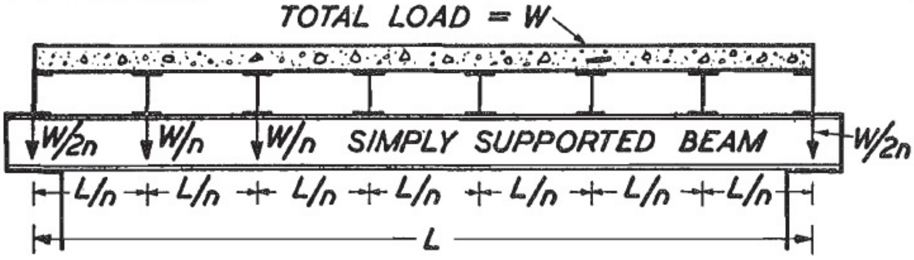
$R_A = R_B = (n-1)\,P/2$

When n is odd

$$d_{max.} = \frac{PL^3}{192EI}\left[n-\frac{1}{n}\right]\left[3-\frac{1}{2}\left(1-\frac{1}{n^2}\right)\right]$$

When n is even

$$d_{max.} = \frac{PL^3}{192EI}\cdot n\left[3-\frac{1}{2}\left(1+\frac{4}{n^2}\right)\right]$$



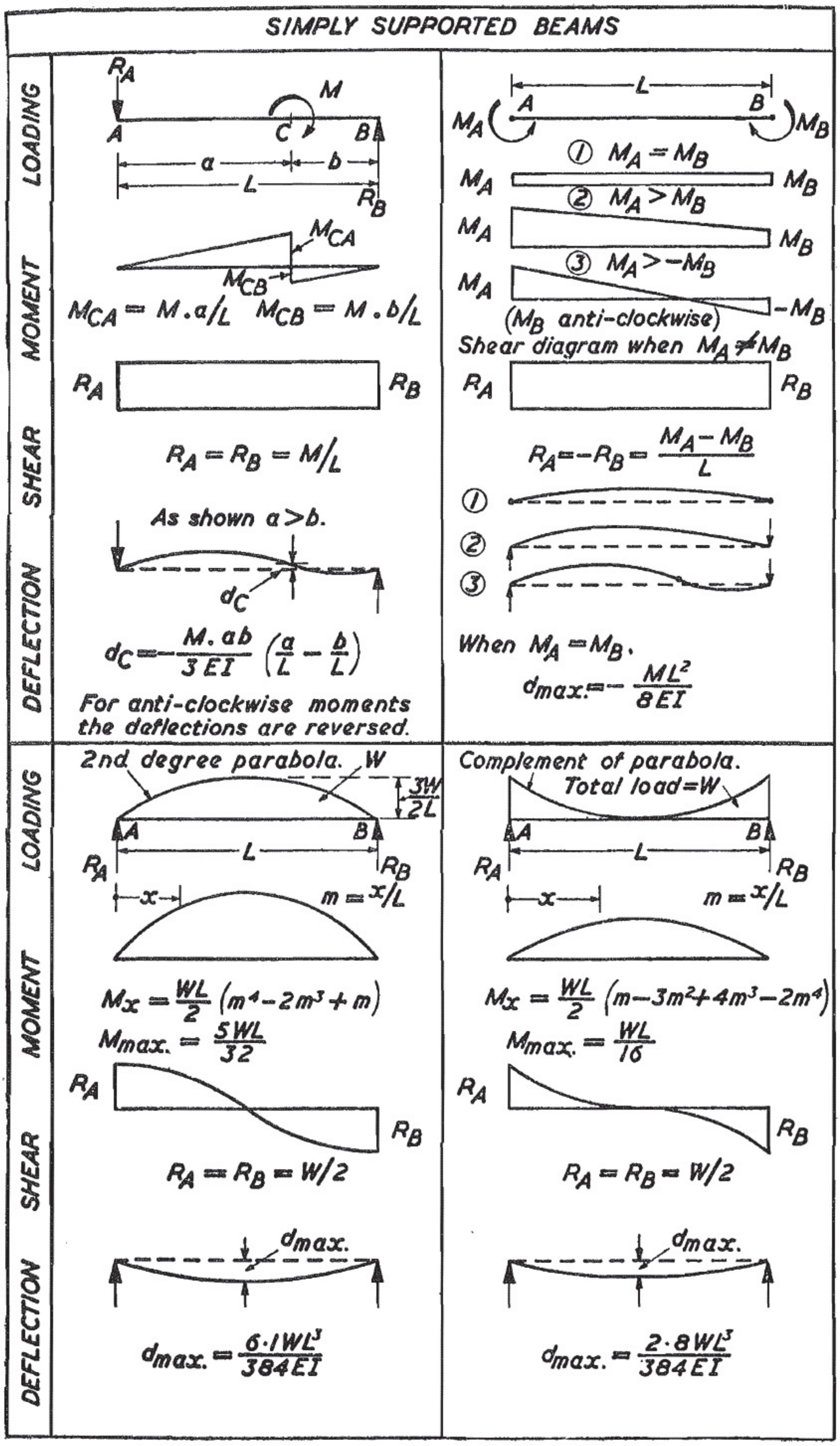
When $n > 10$, consider the load uniformly distributed

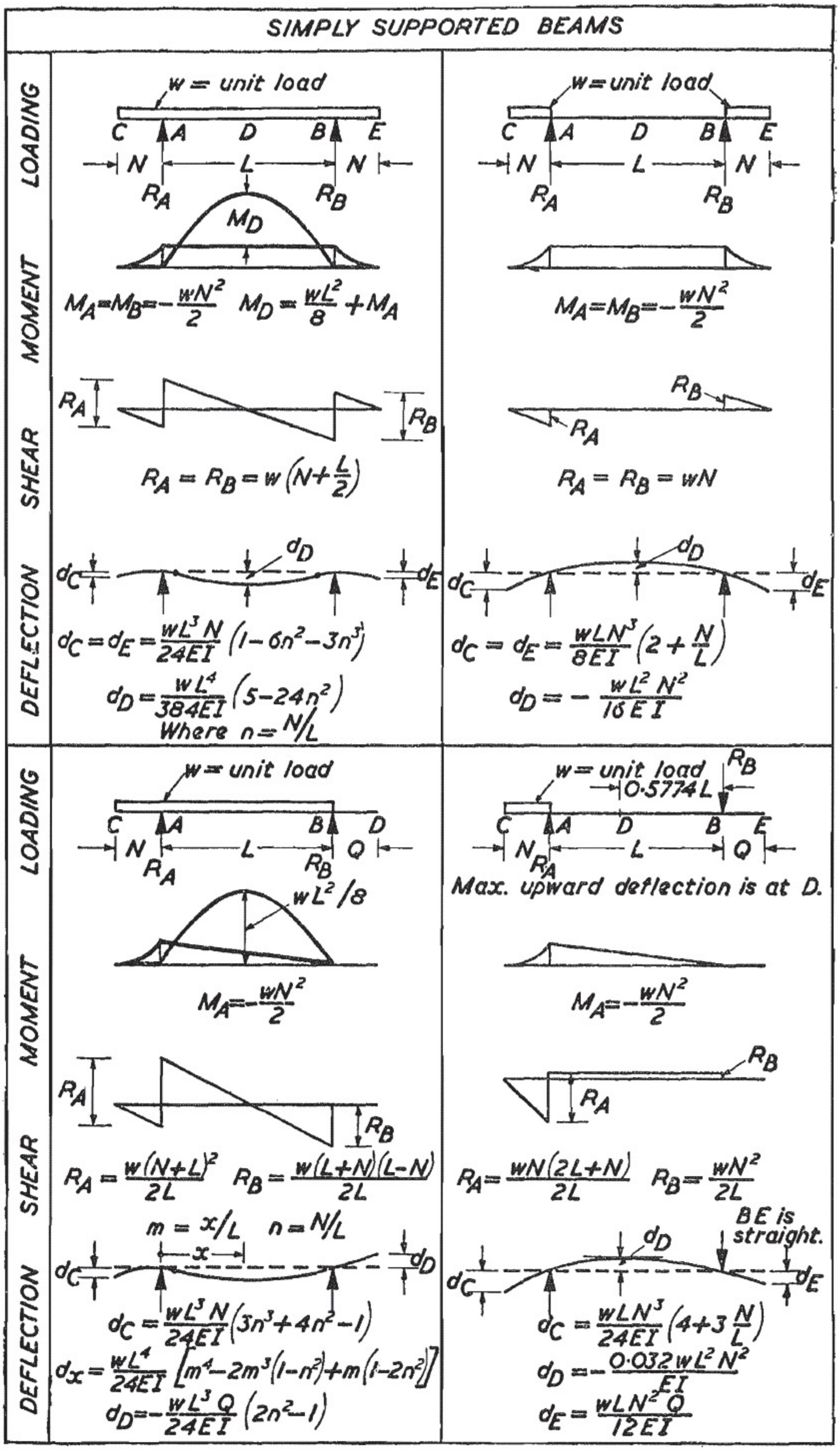
The reaction at the supports = W/2, but the maximum S.F. at the ends of the beam $= \frac{W(n-1)}{2n} = A.W$

The value of the maximum bending moment = C.WL

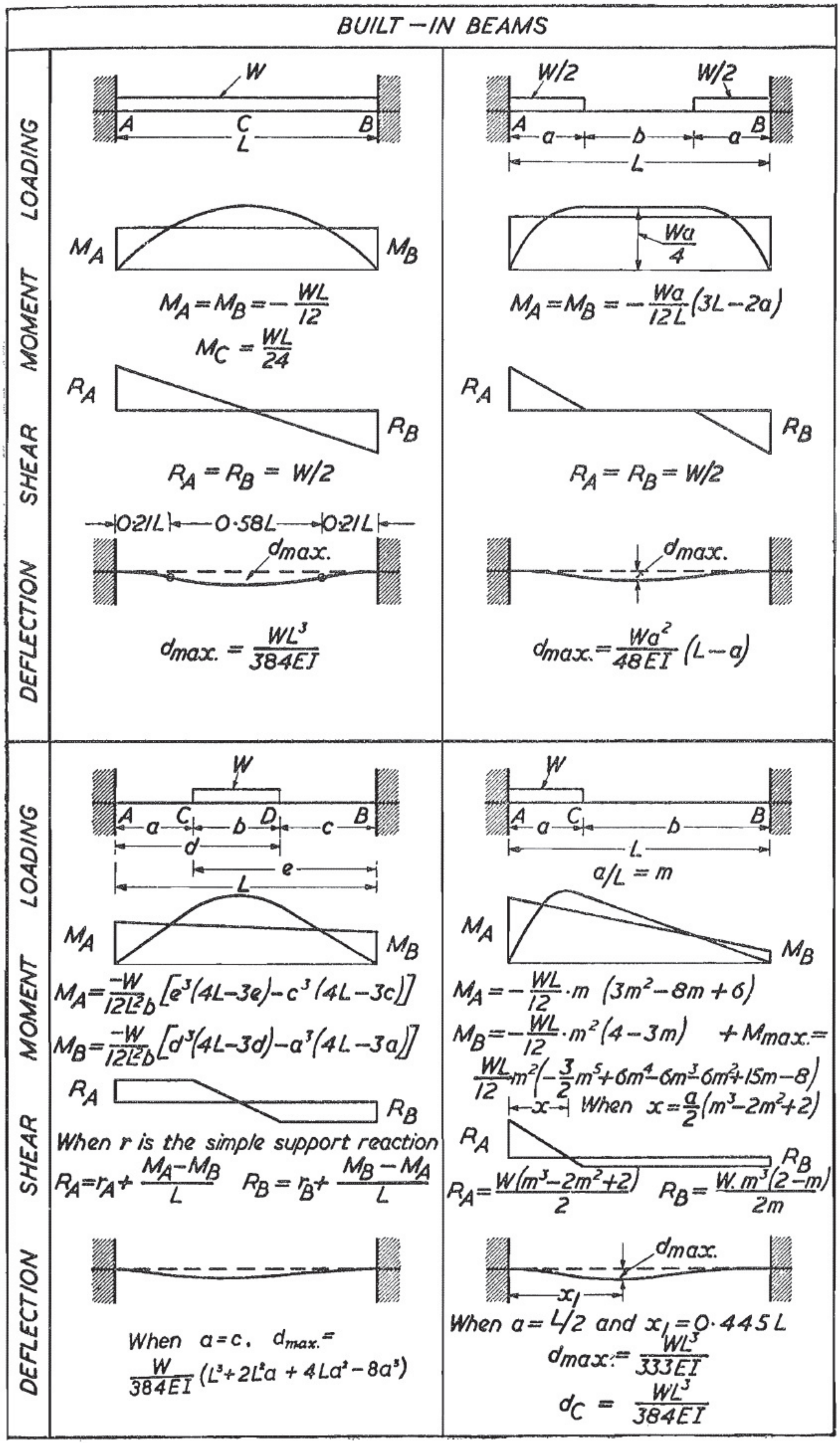
The value of the deflection at the centre of the span $= k.\frac{WL^3}{EI}$

Value of n	A	C	k
2	0·2500	0·1250	0·0105
3	0·3333	0·1111	0·0118
4	0·3750	0·1250	0·0124
5	0·4000	0·1200	0·0126
6	0·4167	0·1250	0·0127
7	0·4286	0·1224	0·0128
8	0·4375	0·1250	0·0128
9	0·4444	0·1236	0·0129
10	0·4500	0·1250	0·0129

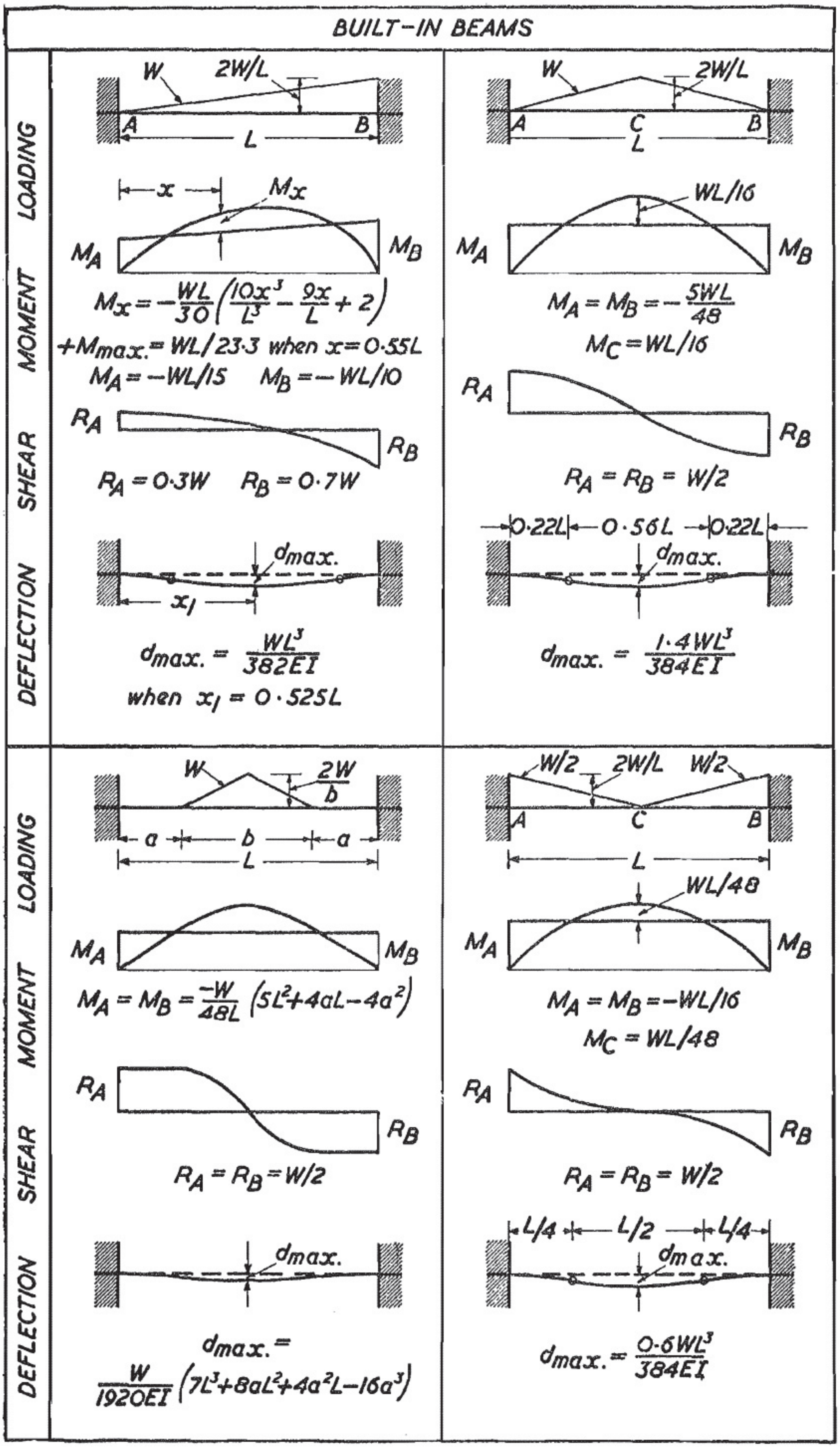

SIMPLY SUPPORTED BEAMS
LOADING
MOMENT
SHEAR
DEFLECTION
RA
M
A
C
B
a
b
L
RB
MCA
MCB
MCA = M.a/L MCB = M.b/L
RA = RB = M/L
As shown a>b.
dC
dC = − M.ab/3EI (a/L − b/L)
For anti-clockwise moments the deflections are reversed.
L
MA
MB
① MA = MB
② MA > MB
③ MA > −MB
−MB
(MB anti-clockwise)
Shear diagram when MA ≠ MB
RA = −RB = (MA − MB)/L
When MA = MB.
dmax. = − ML²/8EI
2nd degree parabola. W
3W/2L
x
m = x/L
Mx = WL/2 (m⁴ − 2m³ + m)
Mmax. = 5WL/32
RA = RB = W/2
dmax.
dmax. = 6·1WL³/384EI
Complement of parabola.
Total load = W
Mx = WL/2 (m − 3m² + 4m³ − 2m⁴)
Mmax. = WL/16
dmax. = 2·8WL³/384EI

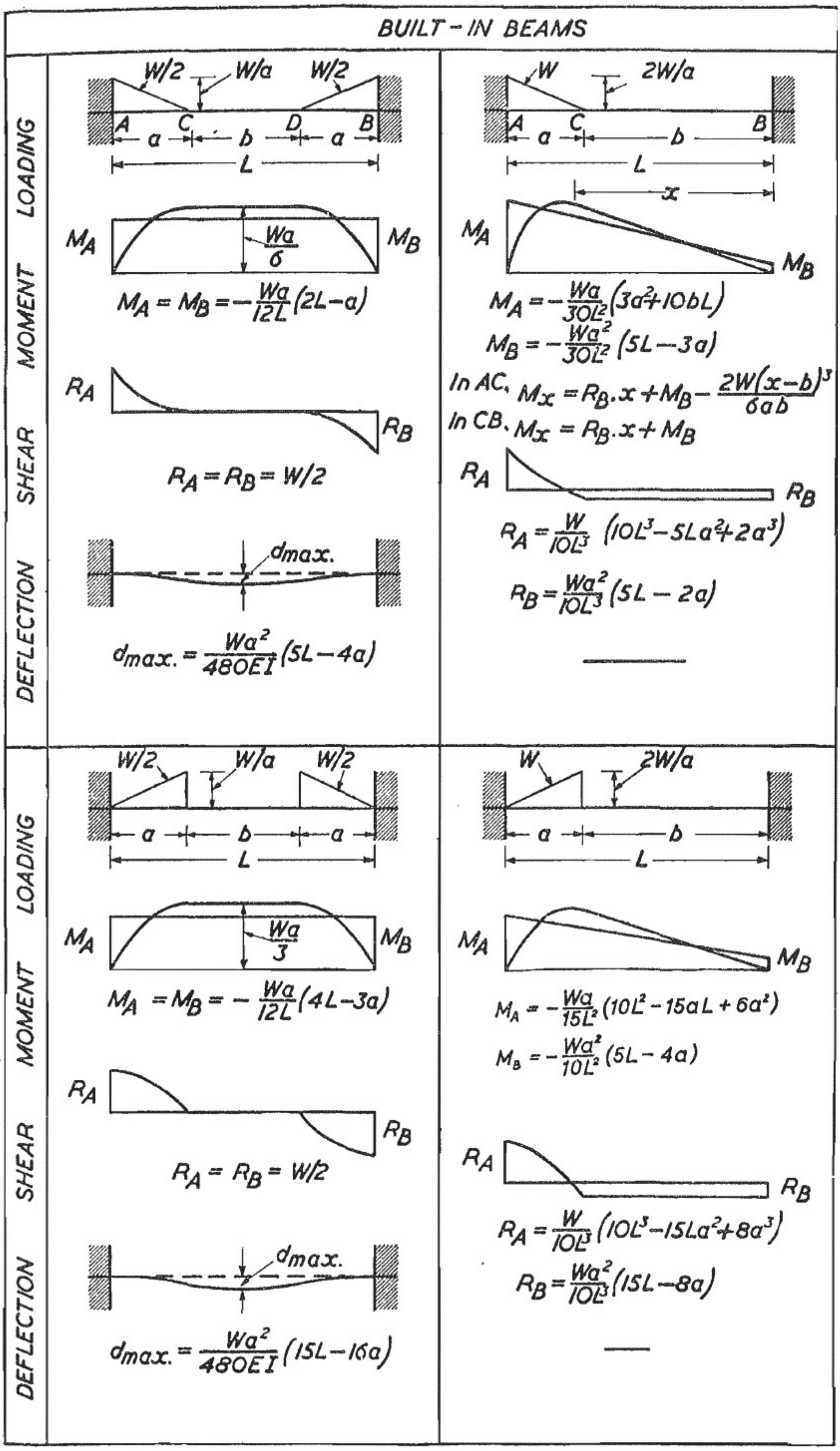


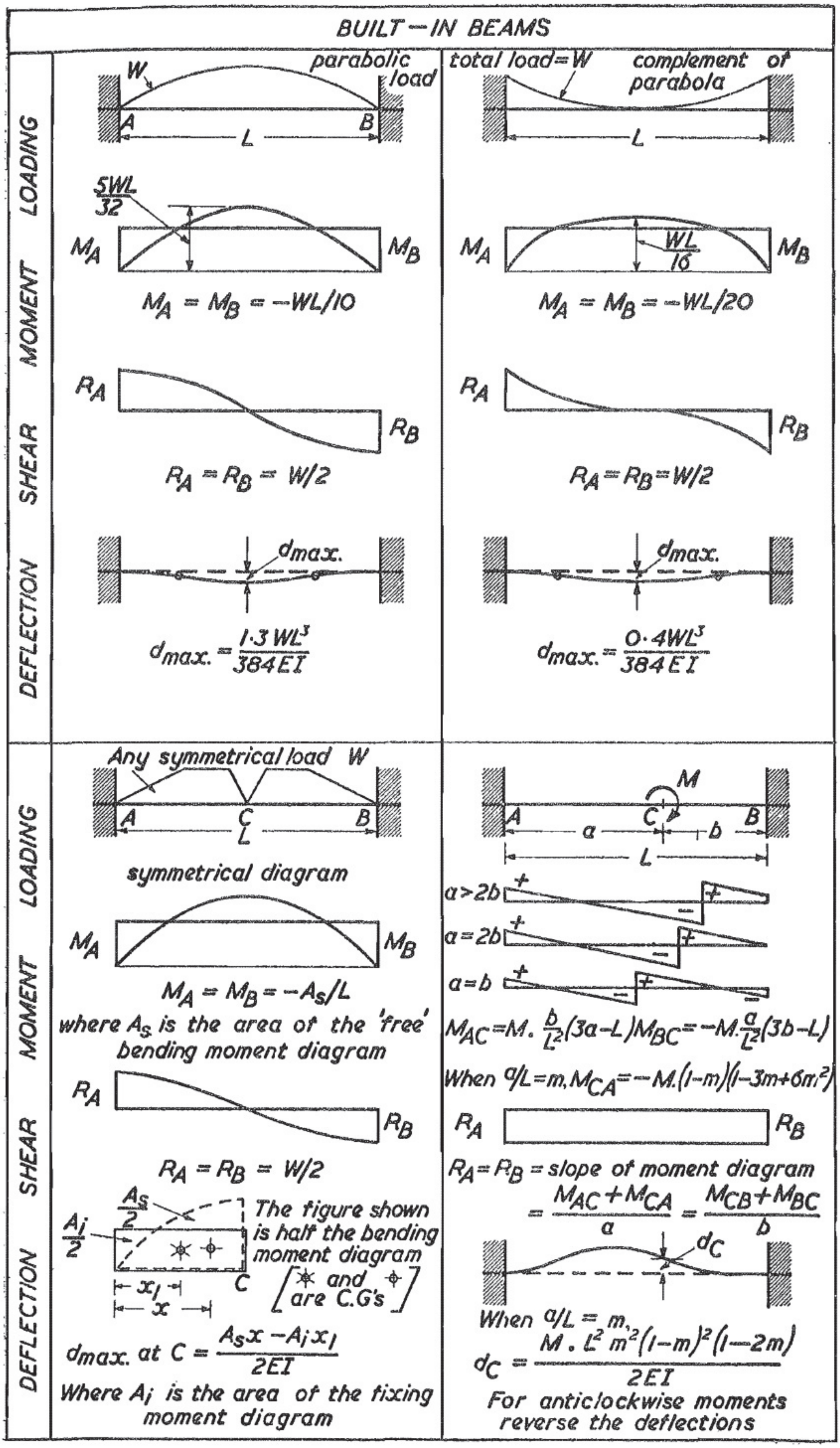
SIMPLY SUPPORTED BEAMS
LOADING
MOMENT
SHEAR
DEFLECTION
w = unit load
C A D B E
N L N
R_A R_B
M_D
$M_A = M_B = -\frac{wN^2}{2}$ $M_D = \frac{wL^2}{8} + M_A$
$R_A = R_B = w\left(N + \frac{L}{2}\right)$
d_C d_D d_E
$d_C = d_E = \frac{wL^3N}{24EI}\left(1 - 6n^2 - 3n^3\right)$
$d_D = \frac{wL^4}{384EI}\left(5 - 24n^2\right)$
Where $n = N/L$
w = unit load
C A D B E
N L N
R_A R_B
$M_A = M_B = -\frac{wN^2}{2}$
$R_A = R_B = wN$
$d_C = d_E = \frac{wLN^3}{8EI}\left(2 + \frac{N}{L}\right)$
$d_D = -\frac{wL^2N^2}{16EI}$
w = unit load
C A B D
N L Q
R_A R_B
$wL^2/8$
$M_A = -\frac{wN^2}{2}$
$R_A = \frac{w(N+L)^2}{2L}$ $R_B = \frac{w(L+N)(L-N)}{2L}$
$m = x/L$ $n = N/L$
d_C x d_D
$d_C = \frac{wL^3N}{24EI}\left(3n^3 + 4n^2 - 1\right)$
$d_x = \frac{wL^4}{24EI}\left[m^4 - 2m^3(1-n^2) + m(1-2n^2)\right]$
$d_D = -\frac{wL^3Q}{24EI}\left(2n^2 - 1\right)$
w = unit load
R_B
0.5774 L
C A D B E
N L Q
R_A
Max. upward deflection is at D.
$M_A = -\frac{wN^2}{2}$
$R_A = \frac{wN(2L+N)}{2L}$ $R_B = \frac{wN^2}{2L}$
BE is straight.
d_C d_D d_E
$d_C = \frac{wLN^3}{24EI}\left(4 + 3\frac{N}{L}\right)$
$d_D = -\frac{0.032\,wL^2N^2}{EI}$
$d_E = \frac{wLN^2Q}{12EI}$

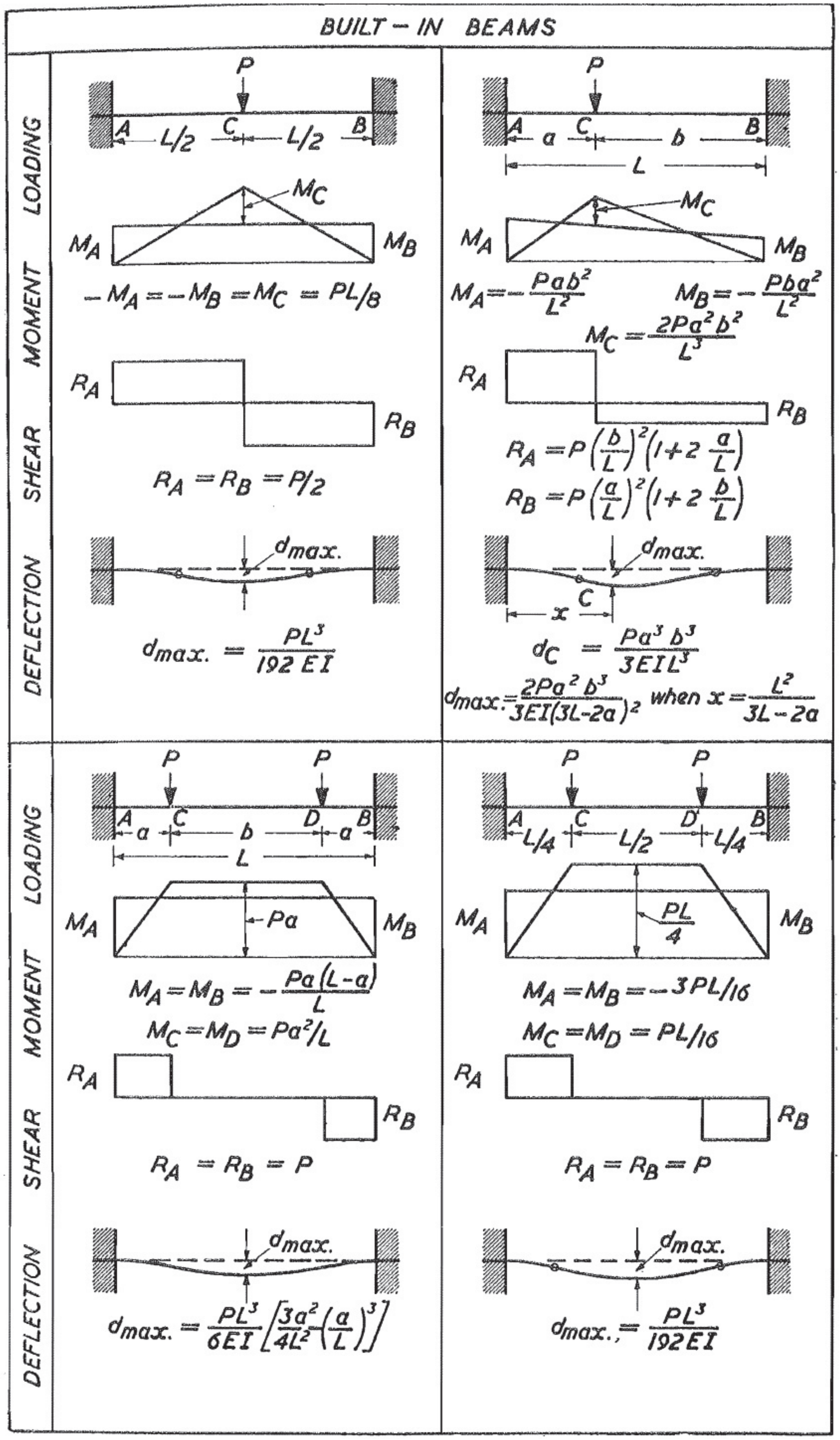


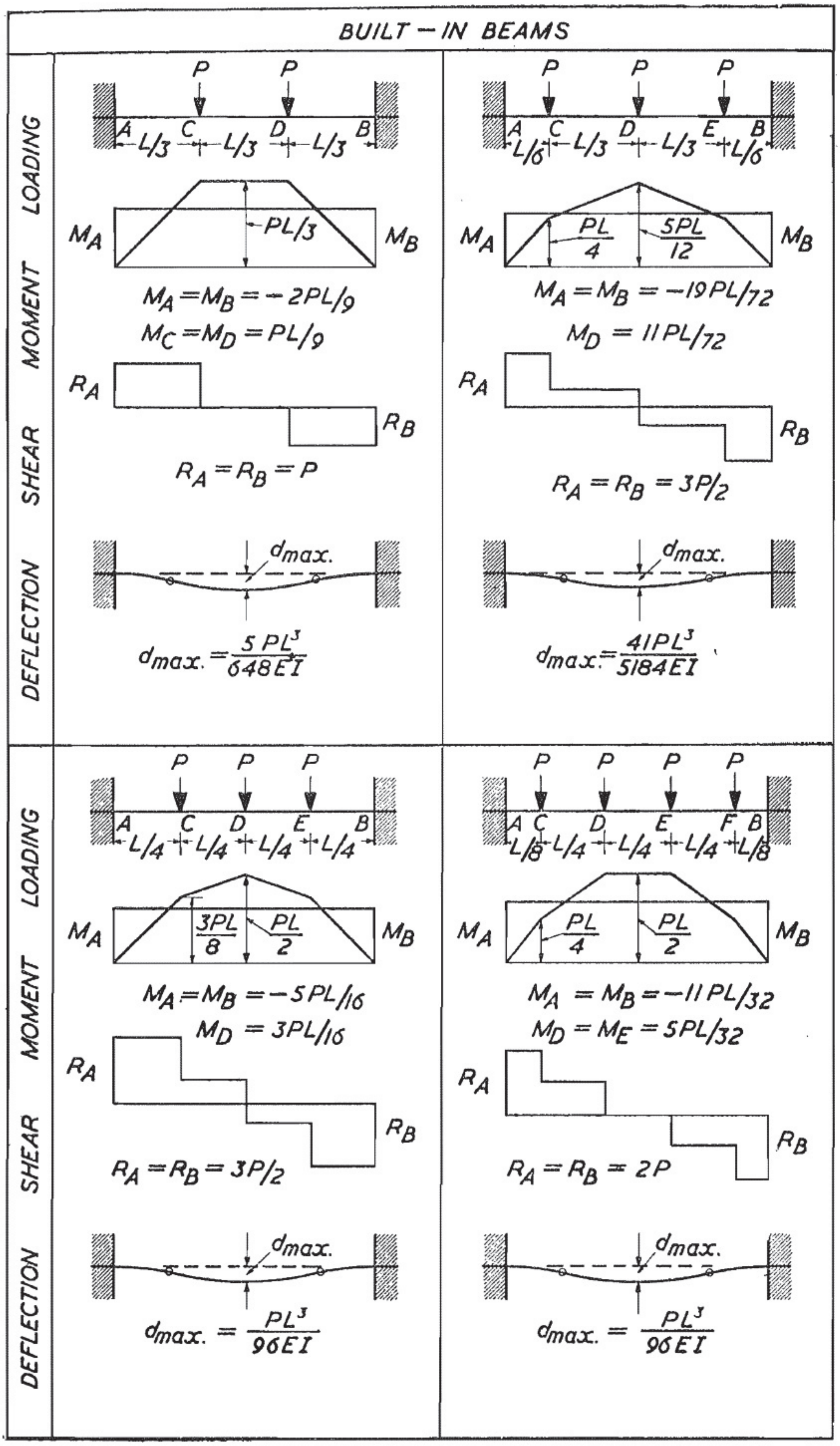
BUILT — IN BEAMS
LOADING
MOMENT
SHEAR
DEFLECTION
W
A C B
L
M_A M_B
$M_A = M_B = -\frac{WL}{12}$
$M_C = \frac{WL}{24}$
R_A R_B
$R_A = R_B = W/2$
0·21L 0·58L 0·21L
$d_{max.}$
$d_{max.} = \frac{WL^3}{384EI}$
W/2 W/2
A a b a B
L
$\frac{Wa}{4}$
$M_A = M_B = -\frac{Wa}{12L}(3L-2a)$
R_A R_B
$R_A = R_B = W/2$
$d_{max.}$
$d_{max.} = \frac{Wa^2}{48EI}(L-a)$
W
A a C b D c B
d
e
L
M_A M_B
$M_A = \frac{-W}{12L^2b}\left[e^3(4L-3e)-c^3(4L-3c)\right]$
$M_B = \frac{-W}{12L^2b}\left[d^3(4L-3d)-a^3(4L-3a)\right]$
R_A R_B
When r is the simple support reaction
$R_A = r_A + \frac{M_A - M_B}{L}$ $R_B = r_B + \frac{M_B - M_A}{L}$
When $a=c$, $d_{max.} = \frac{W}{384EI}(L^3+2L^2a+4La^2-8a^3)$
W
A a C b B
L
$a/L = m$
M_A M_B
$M_A = -\frac{WL}{12}\cdot m\ (3m^2-8m+6)$
$M_B = -\frac{WL}{12}\cdot m^2(4-3m)$ $+M_{max.} = \frac{WL}{12}m^2\left(-\frac{3}{2}m^5+6m^4-6m^3-6m^2+15m-8\right)$
x When $x = \frac{a}{2}(m^3-2m^2+2)$
R_A R_B
$R_A = \frac{W(m^3-2m^2+2)}{2}$ $R_B = \frac{W.m^3(2-m)}{2m}$
$d_{max.}$
x_1
When $a = L/2$ and $x_1 = 0·445L$
$d_{max.} = \frac{WL^3}{333EI}$
$d_C = \frac{WL^3}{384EI}$


BUILT-IN BEAMS
LOADING
MOMENT
SHEAR
DEFLECTION
W
2W/L
A
B
L
x
Mx
MA
MB
Mx = −WL/30 (10x³/L³ − 9x/L + 2)
+Mmax. = WL/23.3 when x = 0.55L
MA = −WL/15   MB = −WL/10
RA
RB
RA = 0.3W   RB = 0.7W
dmax.
x1
dmax. = WL³/382EI
when x1 = 0.525L
W
2W/L
A
C
B
L
WL/16
MA
MB
MA = MB = −5WL/48
MC = WL/16
RA
RB
RA = RB = W/2
0.22L
0.56L
0.22L
dmax.
dmax. = 1.4WL³/384EI
W
2W/b
a
b
a
L
MA
MB
MA = MB = −W/48L (5L²+4aL−4a²)
RA
RB
RA = RB = W/2
dmax.
dmax. = W/1920EI (7L³+8aL²+4a²L−16a³)
W/2
2W/L
W/2
A
C
B
L
WL/48
MA
MB
MA = MB = −WL/16
MC = WL/48
RA
RB
RA = RB = W/2
L/4
L/2
L/4
dmax.
dmax. = 0.6WL³/384EI

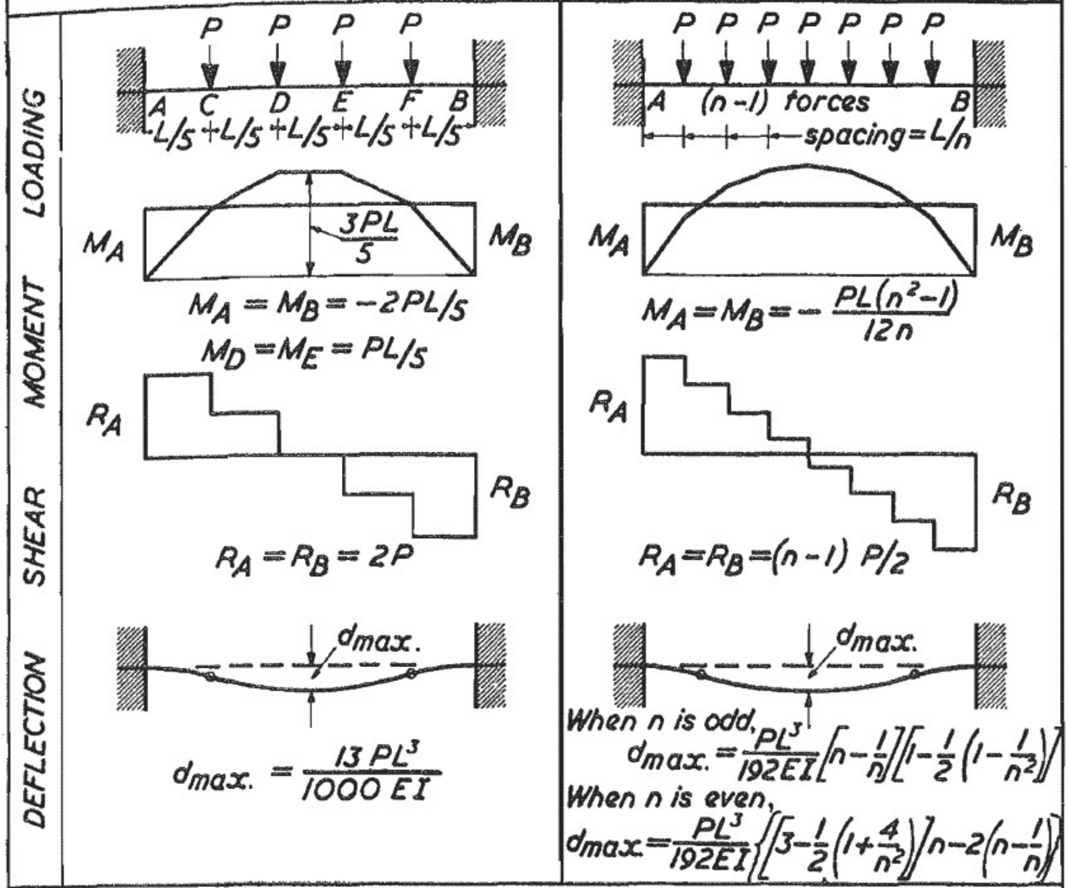

BUILT – IN BEAMS
LOADING
MOMENT
SHEAR
DEFLECTION
W/2
W/a
W/2
A C D B
a b a
L
MA
MB
Wa/6
MA = MB = – Wa/12L (2L–a)
RA
RB
RA = RB = W/2
dmax.
dmax. = Wa²/480EI (5L–4a)
W
2W/a
A C B
a b
L
x
MA
MB
MA = – Wa/30L² (3a²+10bL)
MB = – Wa²/30L² (5L–3a)
In AC, Mx = RB.x + MB – 2W(x–b)³/6ab
In CB, Mx = RB.x + MB
RA
RB
RA = W/10L³ (10L³–5La²+2a³)
RB = Wa²/10L³ (5L – 2a)
W/2
W/a
W/2
a b a
L
MA
MB
Wa/3
MA = MB = – Wa/12L (4L–3a)
RA
RB
RA = RB = W/2
dmax.
dmax. = Wa²/480EI (15L–16a)
W
2W/a
a b
L
MA
MB
MA = – Wa/15L² (10L² – 15aL + 6a²)
MB = – Wa²/10L² (5L – 4a)
RA
RB
RA = W/10L³ (10L³–15La²+8a³)
RB = Wa²/10L³ (15L–8a)

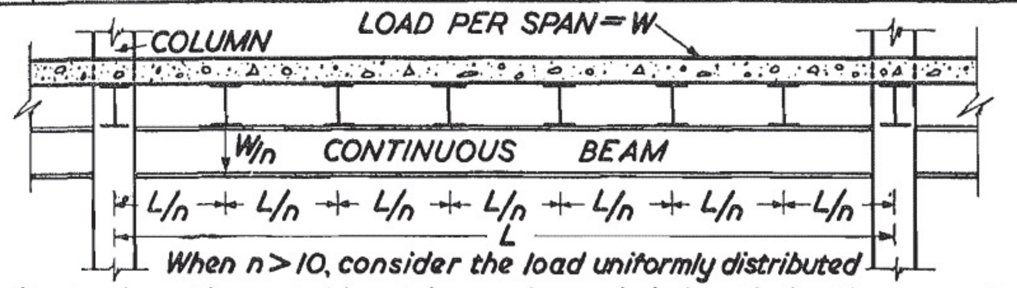

BUILT — IN BEAMS
LOADING
MOMENT
SHEAR
DEFLECTION
W
parabolic load
A
B
L
5WL/32
MA
MB
MA = MB = —WL/10
RA
RB
RA = RB = W/2
dmax.
dmax. = 1·3 WL³/384EI
total load=W
complement of parabola
L
WL/16
MA = MB = —WL/20
RA = RB = W/2
dmax.
dmax. = 0·4WL³/384EI
Any symmetrical load W
A
C
B
L
symmetrical diagram
MA = MB = —As/L
where As is the area of the 'free' bending moment diagram
RA = RB = W/2
As/2
Ai/2
The figure shown is half the bending moment diagram
[✳ and ⌖ are C.G's]
C
x1
x
dmax. at C = (As x − Ai x1)/2EI
Where Ai is the area of the fixing moment diagram
M
A
C
B
a
b
L
a>2b
a=2b
a=b
+
−
MAC = M. b/L²(3a−L) MBC = −M. a/L²(3b−L)
When a/L = m, MCA = −M.(1−m)(1−3m+6m²)
RA = RB = slope of moment diagram
= (MAC + MCA)/a = (MCB + MBC)/b
dC
When a/L = m,
dC = M . L² m²(1−m)²(1−2m)/2EI
For anticlockwise moments reverse the deflections


BUILT - IN BEAMS
LOADING
MOMENT
SHEAR
DEFLECTION
P
A C B
L/2 L/2
M_C
M_A M_B
$-M_A = -M_B = M_C = PL/8$
R_A R_B
$R_A = R_B = P/2$
$d_{max.}$
$d_{max.} = \frac{PL^3}{192\,EI}$
P
A C B
a b
L
M_C
M_A M_B
$M_A = -\frac{Pab^2}{L^2}$
$M_B = -\frac{Pba^2}{L^2}$
$M_C = \frac{2Pa^2b^2}{L^3}$
R_A R_B
$R_A = P\left(\frac{b}{L}\right)^2\left(1+2\,\frac{a}{L}\right)$
$R_B = P\left(\frac{a}{L}\right)^2\left(1+2\,\frac{b}{L}\right)$
$d_{max.}$
x C
$d_C = \frac{Pa^3b^3}{3EIL^3}$
$d_{max.} = \frac{2Pa^2b^3}{3EI(3L-2a)^2}$ when $x = \frac{L^2}{3L-2a}$
P P
A C D B
a b a
L
Pa
M_A M_B
$M_A = M_B = -\frac{Pa(L-a)}{L}$
$M_C = M_D = Pa^2/L$
R_A R_B
$R_A = R_B = P$
$d_{max.}$
$d_{max.} = \frac{PL^3}{6EI}\left[\frac{3a^2}{4L^2} - \left(\frac{a}{L}\right)^3\right]$
P P
A C D B
L/4 L/2 L/4
$\frac{PL}{4}$
M_A M_B
$M_A = M_B = -3PL/16$
$M_C = M_D = PL/16$
R_A R_B
$R_A = R_B = P$
$d_{max.}$
$d_{max.} = \frac{PL^3}{192EI}$


BUILT — IN BEAMS
LOADING
MOMENT
SHEAR
DEFLECTION
P P
A C D B
L/3 L/3 L/3
PL/3
M_A M_B
M_A = M_B = − 2PL/9
M_C = M_D = PL/9
R_A R_B
R_A = R_B = P
d_max.
d_max. = 5 PL³ / 648 EI
P P P
A C D E B
L/6 L/3 L/3 L/6
PL/4 5PL/12
M_A M_B
M_A = M_B = −19 PL/72
M_D = 11 PL/72
R_A R_B
R_A = R_B = 3P/2
d_max.
d_max. = 41 PL³ / 5184 EI
P P P
A C D E B
L/4 L/4 L/4 L/4
3PL/8 PL/2
M_A M_B
M_A = M_B = − 5PL/16
M_D = 3PL/16
R_A R_B
R_A = R_B = 3P/2
d_max.
d_max. = PL³ / 96 EI
P P P P
A C D E F B
L/8 L/4 L/4 L/4 L/8
PL/4 PL/2
M_A M_B
M_A = M_B = −11 PL/32
M_D = M_E = 5PL/32
R_A R_B
R_A = R_B = 2P
d_max.
d_max. = PL³ / 96 EI

BUILT – IN BEAMS





When $n > 10$, consider the load uniformly distributed

The load on the outside stringers is carried directly by the supports

The continuous beam is assumed to be horizontal at each support

The reaction at the supports for each span $= W/2$, but the maximum shear force in any span of the continuous beam $= \frac{W(n-1)}{2n} = A.W$

The value of the fixing moment at each support $= -\,B.WL$

The value of the maximum positive moment for each span $= C.WL$

The value of the maximum deflection for each span $\simeq 0{\cdot}0026\,\frac{WL^3}{EI}$

Value of n	A	B	C
2	0·2500	0·0625	0·0625
3	0·3333	0·0741	0·0370
4	0·3750	0·0781	0·0469
5	0·4000	0·0800	0·0400
6	0·4167	0·0811	0·0439
7	0·4286	0·0816	0·0408
8	0·4375	0·0820	0·0430
9	0·4444	0·0823	0·0413
10	0·4500	0·0825	0·0425